Subspace Outlier Detection in Linear Time with Randomized Hashing

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Motivation – Fast and simple subspace outlier detection

- **Simplicity:** Simple to implement $\Longrightarrow$ requires few lines of code to implement
- **Accuracy:** Accurate for high-dimensional data sets
- **Linear Complexity:** Linear in the number of points $\Longrightarrow$ orders of magnitude speedup
- **Constant Memory:** Constant memory $\Longrightarrow$ memory requirement does not change with the number of points
- **Stream Compatibility:** Works for data streams $\Longrightarrow$ on-the-fly outlier scoring is possible

RS-Hash: Randomized Subspace Hashing

RS-Hash is ensemble centric $\Longrightarrow$

- It is comprised of several weak outlier detectors
- Each individual detector is extremely weak, but the overall performance is extraordinarily good.

How does it work?

- **Input:** Data set D with n points and d dimensions. The ith row is a d-dimensional vector denoted by $\overline{X}_i = (x_{i1}...x_{id})$.
- **Output:** Outlier score for each point in D.

Steps:

1. Sample s points from the data set D.
2. Learn a training model on these s points
3. Score all the data points using this training model
4. Perform steps 1-3 for m times and report the average score

RS-Hash: Randomized Subspace Hashing

Steps for A Single Ensemble Component

1. Select a value of the locality parameter f randomly between $(1/\sqrt{s}, 1 - 1/\sqrt{s})$.

2. Generate a random vector $(\alpha_1 \ldots \alpha_d)$, where α_i is drawn randomly from $(0, f)$. α_i is referred to as the shift parameter.

3. Select an integer r randomly between $1 + 0.5 \cdot [\log_{\max\{2,1/f\}}(s)]$ and $\log_{\max\{2,1/f\}}(s)$. Select r dimensions and denote this set by V.

4. Draw a training sample S of s points and determine the min and max values (min_j and max_j) for each dimension j

5. Normalize each $\overline{X_i}$ in the sample to $\overline{X_i'}$ as follows:

$$x'_{ij} \Leftarrow \frac{x_{ij} - min_j}{max_j - min_j} \tag{1}$$

RS-Hash: Randomized Subspace Hashing

6. For each $\overline{X'_i}$ create its *integer representation* $\overline{Y}_i$ such that:

- if $j \notin V$ set y_{ij} to -1
- if $j \in V$ set y_{ij} to the integer value of $\lfloor (x'_{ij} + \alpha_j)/f \rfloor$.

8. Apply w different hash functions $h_1(\overline{Y}_i) \ldots h_w(\overline{Y}_i)$ to $\overline{Y}_i$. Increment the $h_k(\overline{Y}_i)$th element of the kth hash table by 1.

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Scoring step:

9. Transform each point $\overline{X} \in D$ to $\overline{Y}$ using the same steps as before. Apply the same w hash function to points $\overline{Y}$.

10. Let the value of the $h_k(\overline{Y})$th cell in the kth hash table be c_k
    - If $\overline{Y} \in S$ report $\log_2(\min\{c_1 \ldots c_w\})$ as the outlier score;
    - Otherwise, report $\log_2(\min\{c_1 \ldots c_w\}+1)$

Implementing the Hash Functions

RS-Hash(S) $\rightarrow$ Streaming Variant $\rightarrow$ Count-min Sketch

- Insert: $\overline{Y}$ is hashed w times. Each hashed output maps to a bucket ID between $(0, p-1)$, whose count is incremented by 1.
- p is the size of hash table and is known as the hash range and is selected based on memory availability
- Lookup: $\overline{Y}$ is again hashed w times. All counts are retrieved and the minimum is returned.
- Typical values of p and w are $p = 10,000$ and $w = 4$
- Collisions occur with very low probability, and (theoretically) any number of elements can be inserted

RS-Hash(E) $\rightarrow$ Static Variant $\rightarrow$ Hash Table with Linear Probing

- This is the standard hash table with linear probing.

Why does the Approach Work?

$$\text{Log-Fit}(\overline{Y}) = \log(\rho(\overline{Y})) = \log\left(\frac{1+\min\{c_1 \ldots c_w\}}{1+n}\right)$$

- Overall score of *RS-Hash* can be written as

$$\text{Score}(\overline{Y}) = \text{Const.} + \text{AVG}_{[\text{All } V]} \log\left(1+\min\{c_1 \ldots c_w\}\right)$$

- *RS-Hash* computes ensemble-centric average log-likelihood fits over different base components

Logic of Parameter Choices

- $f \in [1/\sqrt{s}, (1-1/\sqrt{s})]$. Ensures that larger grids are explored when there are fewer points
- The dimensionality r of the subspace explored is at least 2. This is because the value of r is at least $1 + 0.5\log_{\max\{2,1/f\}}(s) \geq 1 + 0.5\log_{\sqrt{s}}(s) = 2$.
- The dimensionality selected is such that the expected number of points in the local region corresponding to each test point varies between 1 and $\sqrt{s}$.

Experiments – Data sets

Setup: All data sets are from the UCI Machine Learning Repository. Ensemble components $m = 300$, hash tables $w = 4$, hash range $p = 10,000$

Data set	Points	Attribute	Percent Outliers (%)
Lympho	148	18	3.4
Ecoli	336	7	2.7
Yeast	1,364	8	4.8
Cardio	1,831	21	9.6
Musk	3,062	166	3.1
Waveform	3,509	21	4.7
Optdigits	5,216	64	2.9
KDDCup99	25,000	41	0.7

Baselines: LoF [1], FastABOD [2], HiCS [3], iForest [4], and AvgKNN [5]

[1] M. M. Breunig, H.-P. Kriegel, R. T. Ng, and J. Sander. LOF: identifying density-based local outliers. SIGMOD 2000.

[2] H.-P. Kriegel, M. Schubert, and A. Zimek. Angle-based Outlier Detection in High-Dimensional Data. KDD 2008.

[3] F. Keller, E. Muller, K. Bohm. HiCS: High-Contrast Subspaces for Density-based Outlier Ranking. ICDE 2012.

[4] Liu, F. T., Ting, K. M., Zhou, Z. H. Isolation forest. ICDM 2008.

[5] Angiulli, F., Pizzuti, C. Fast outlier detection in high dimensional spaces. PKDD 2002.

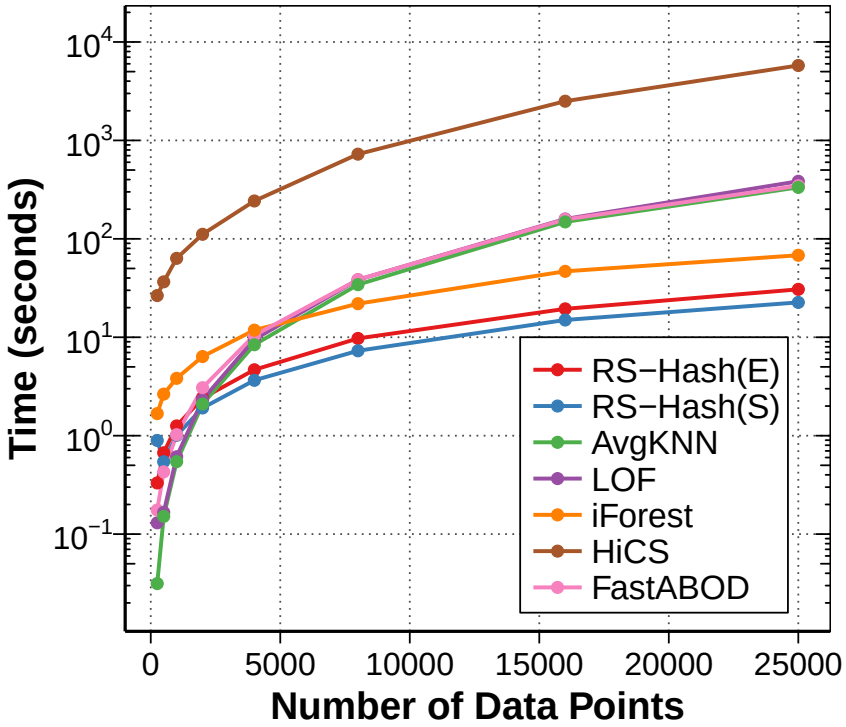
Experiments – Area under curve (AUC)

Data set	RS-Hash(E)	RS-Hash(S)	AvgKNN	LOF	iForest	HiCS	FastABOD
Lympho	**100.0**	**99.85**	98.16	97.18	99.71	86.78	52.32
Ecoli	**88.41**	**88.44**	87.62	86.29	85.31	73.72	84.66
Cardio	**91.61**	**91.78**	70.47	59.67	91.52	53.04	58.74
Musk	**100.0**	**100.0**	24.48	39.99	**100.0**	47.77	23.95
Optdigits	**76.04**	**76.14**	39.59	61.54	72.66	39.05	72.07
Yeast	**80.87**	**80.80**	66.47	55.06	79.44	61.23	66.91
Waveform	**74.42**	**73.85**	66.98	61.08	72.47	62.61	58.91
KDDCup99	**99.97**	**99.98**	13.6	46.43	**99.98**	79.94	13.71

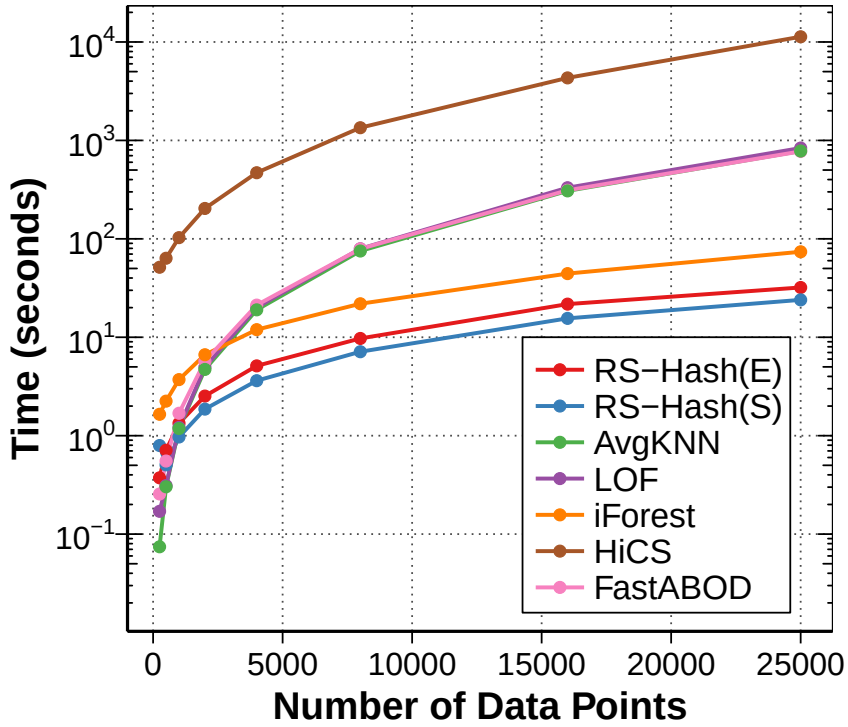
* top-2 methods are shown in boldface

- Highest average improvement of 53% is observed in Musk
- Musk is a very high dimensional data set, which demonstrates the effectiveness of randomized feature selection strategy of *RS-Hash*.
- *RS-Hash* consistently improves on the baselines

Experiments – Efficiency Analysis



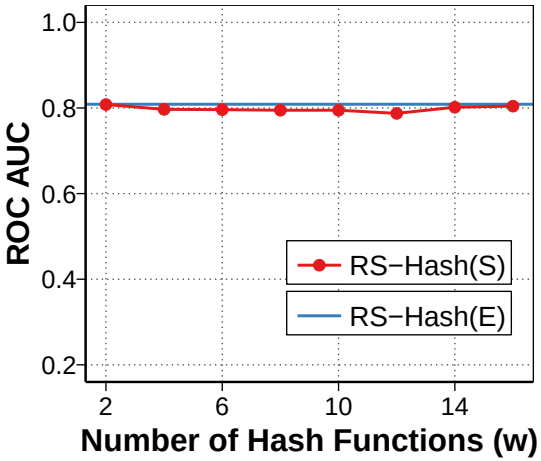
(a) KDDCup99



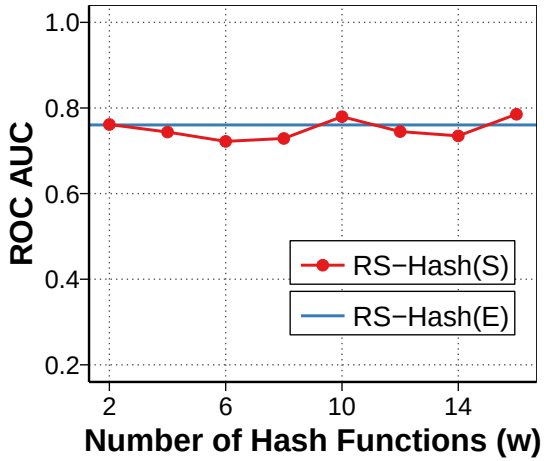
(b) NORMAL(0,1)

- *RS-Hash(S)* is between 20 and 100 times faster than LOF for KDDCup99
- *RS-Hash* is nearly 400 times faster than HiCS at this data size.
- The difference between *RS-Hash* and baselines only increases with data size

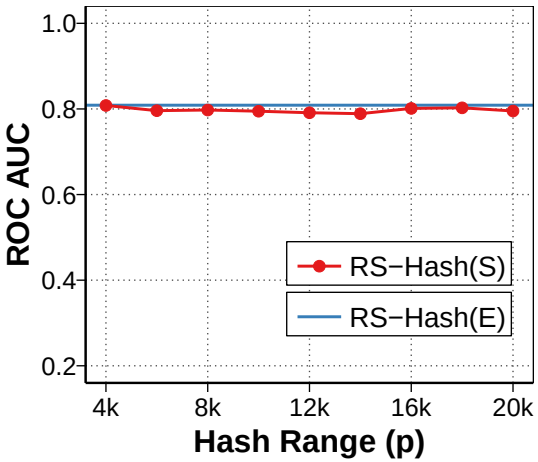
Experiments – Parameter Sensitivity



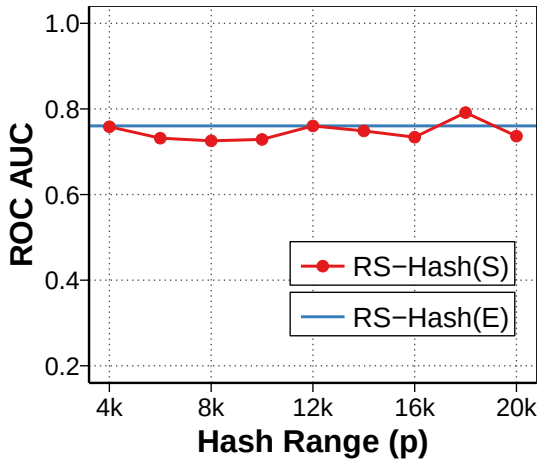
(a) YEAST



(b) OPTDIGITS



(c) YEAST



(d) OPTDIGITS

Additional experiments on parameter sensitivity analysis and the streaming version *RS-Hash(S)* can be found in the paper.

Summary and Conclusion

- We discussed the randomized hashing-based outlier detector
- The discussed methods are simple and easy to implement and only rely on hash tables
- We discussed how to effectively use the min-hash data structure with *RS-Hash*
- Demonstrated accuracy and efficiency of *RS-Hash* with extensive experiments on real data sets

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Thank You.

We are hiring!

Backup

Extensions to Data Streams

Instead initializing each hash function one after another, we initialize them simultaneously.

- Time-decayed scores: The weight of each point is reduced after the arrival of an additional data point
- It becomes $2^{-\lambda t}$ after t points have arrived $\implies$ half-life $= 1/\lambda$
- The values of min_j and max_j are initialized on using an initial sample of data
- Since the data set size is not fixed, we determine $s = \max\{1000, \sum_i 2^{-\lambda i}\} = \max 1000, \frac{1}{1-2^{-\lambda}}$

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- Since the data set size is not fixed, we determine $s = \max\{1000, \sum_i 2^{-\lambda i}\} = \max 1000, \frac{1}{1-2^{-\lambda}}$
- Access: count $* 2^{-\lambda(t_c - t_l)}$
- Update: new count $= 2^{-\lambda(t_c - t_l)} *$ old count $+ 1$
- t_c current time stamp, t_l last time stamp